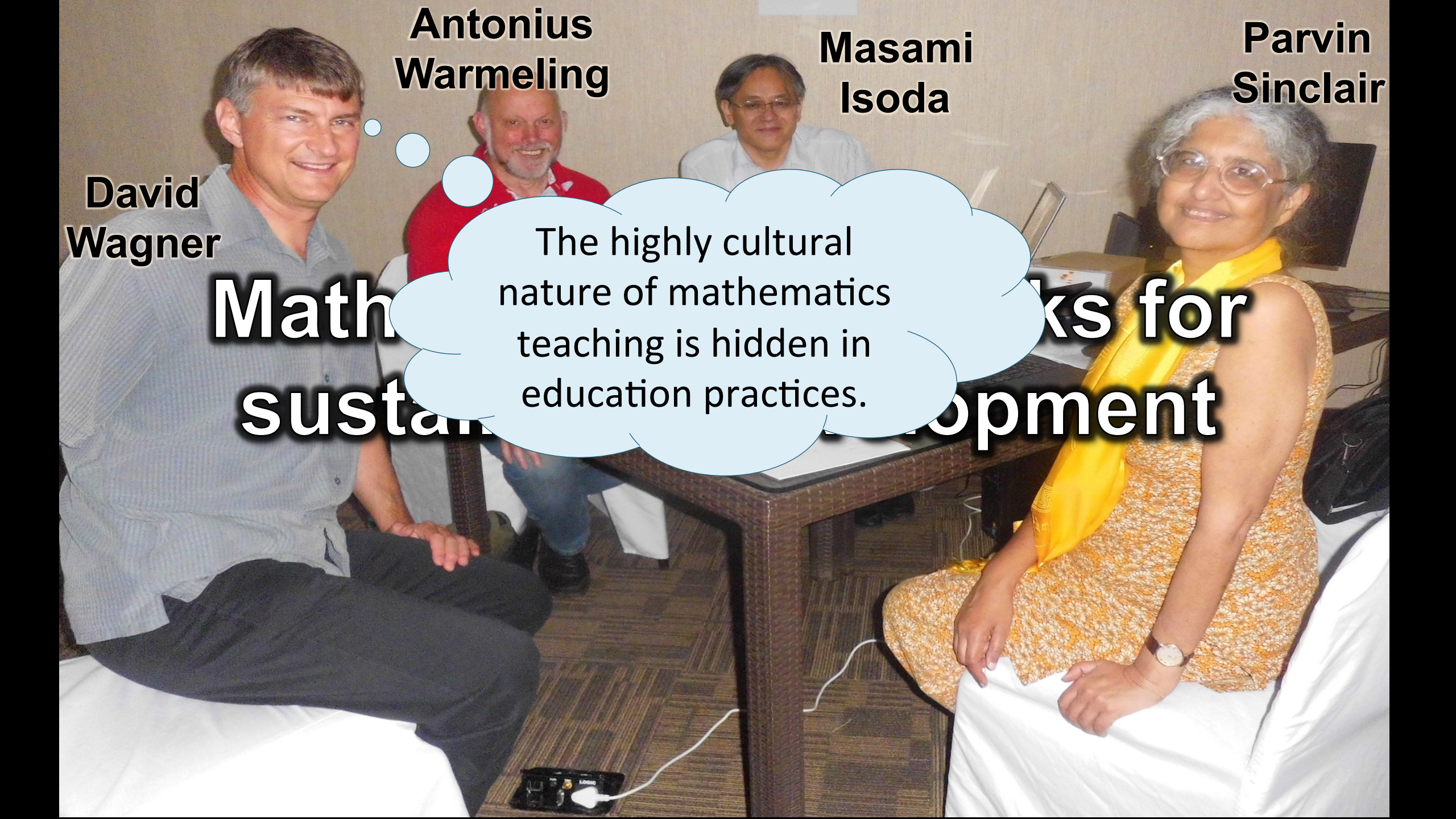


Mathematics textbooks for sustainable development

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Antonius
Warmeling

Masami
Isoda

Parvin
Sinclair

David
Wagner

Maths for sustainable development

The highly cultural nature of mathematics teaching is hidden in education practices.

Sustainable Development Goals (SDGs)



Sustainable Development Goals (SDGs)

1. end poverty in all its forms everywhere

2. end hunger, achieve food security and improved nutrition and promote sustainable agriculture

3. ensure healthy lives and promote well-being for all at all ages

4. ensure inclusive and equitable quality education and promote lifelong learning opportunities for all

5. achieve gender equality and empower all women and girls

6. ensure availability and sustainable management of water and sanitation for all

7. ensure access to affordable, reliable, sustainable and modern energy for all

Sustainable Development Goals (SDGs)

8. promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all

9. build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation

10. reduce inequality within and among countries

11. make cities and human settlements inclusive, safe, resilient and sustainable

12. ensure sustainable consumption and production patterns

13. take urgent action to combat climate change and its impacts

14. conserve and sustainably use the oceans, seas and marine resources for sustainable development

Sustainable Development Goals (SDGs)

15. protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss

16. promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels

17. strengthen the means of implementation and revitalize the global partnership for sustainable development



TEXTBOOKS FOR SUSTAINABLE DEVELOPMENT

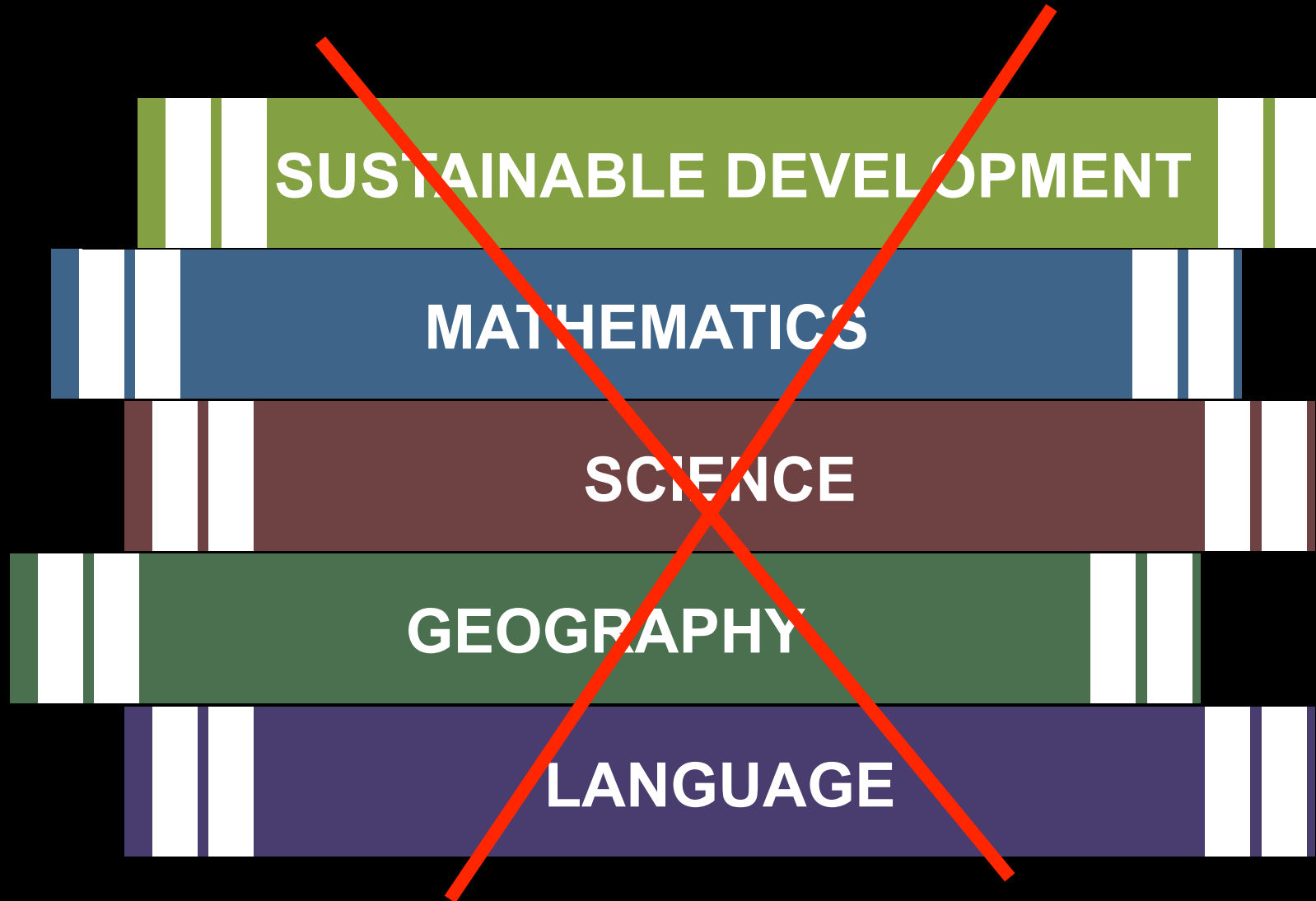
A GUIDE TO EMBEDDING



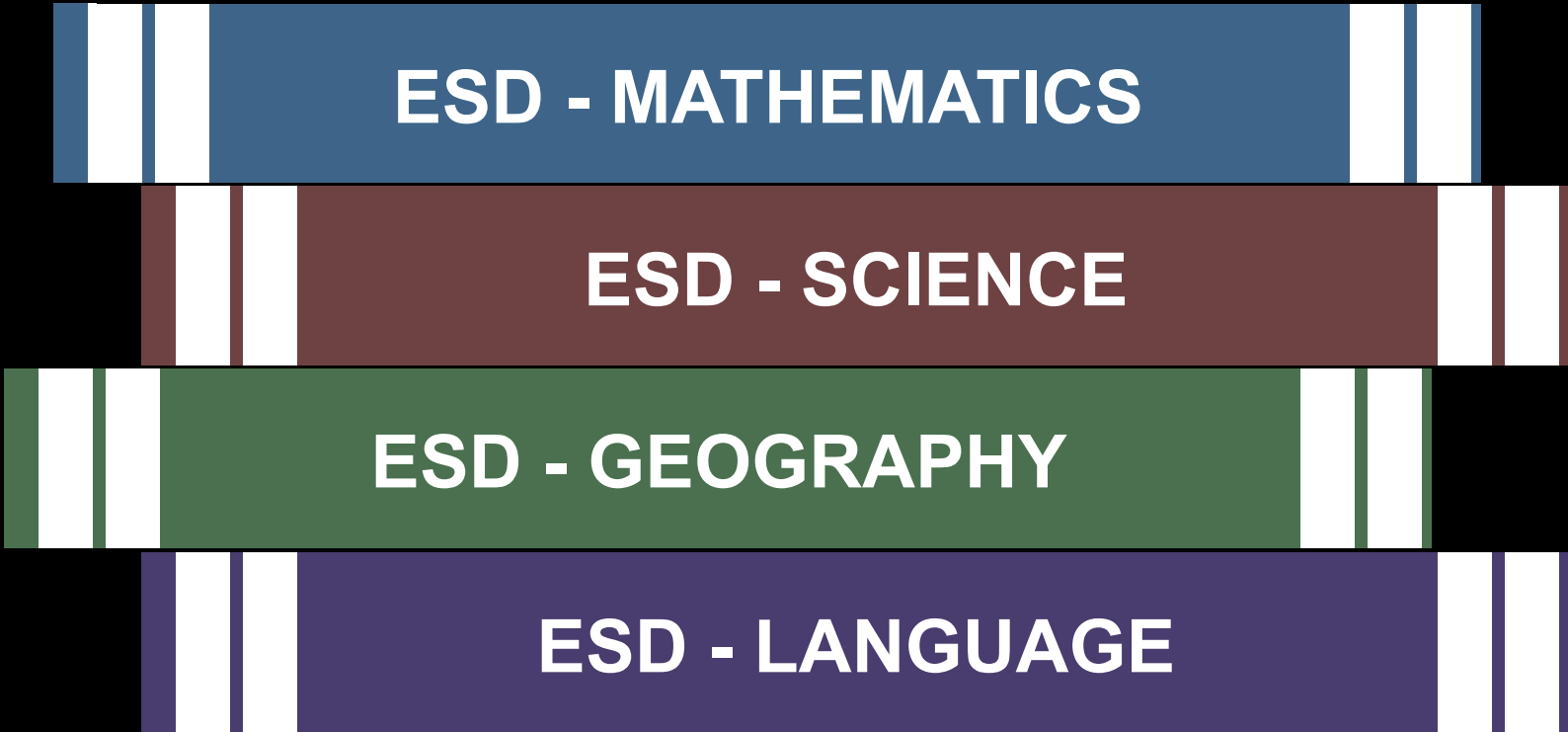
- MATHEMATICS
- SCIENCE
- GEOGRAPHY
- LANGUAGE

<https://tinyurl.com/sustainablemath>
[https tiny url .com sustainable math](https://tinyurl.com/sustainablemath)

Sustainability is not an “add on”. It is the core.



Sustainability is not an “add on”. It is the core.



The image features four horizontal bars of different colors, stacked vertically. Each bar has a central text label and is flanked by two vertical white stripes on each side. The bars are: 1. Blue bar with 'ESD - MATHEMATICS', 2. Maroon bar with 'ESD - SCIENCE', 3. Green bar with 'ESD - GEOGRAPHY', and 4. Purple bar with 'ESD - LANGUAGE'.

ESD - MATHEMATICS

ESD - SCIENCE

ESD - GEOGRAPHY

ESD - LANGUAGE

Potential contributions of mathematics (basis for task/activity development)

Philosophies of mathematics:

- formalism
- platonism
- humanism
- etc.

To teach mathematics for sustainable development a humanist understanding of mathematics is most relevant.

Mathematics is a human activity,
a way of addressing challenges in particular
human contexts.

(or a set of ways)

Potential contributions of mathematics (basis for task/activity development)

Six activities of mathematics (Bishop):

- counting
- measuring
- locating
- designing
- explaining
- playing

When people design, explain, or play with counting, measuring or locating, they are doing mathematics.

Potential contributions of mathematics (basis for task/activity development)

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When people design
they are doing mathematics



measuring or locating,



Ayutthaya, 12th Century

- locating
- designing
- explaining
- playing

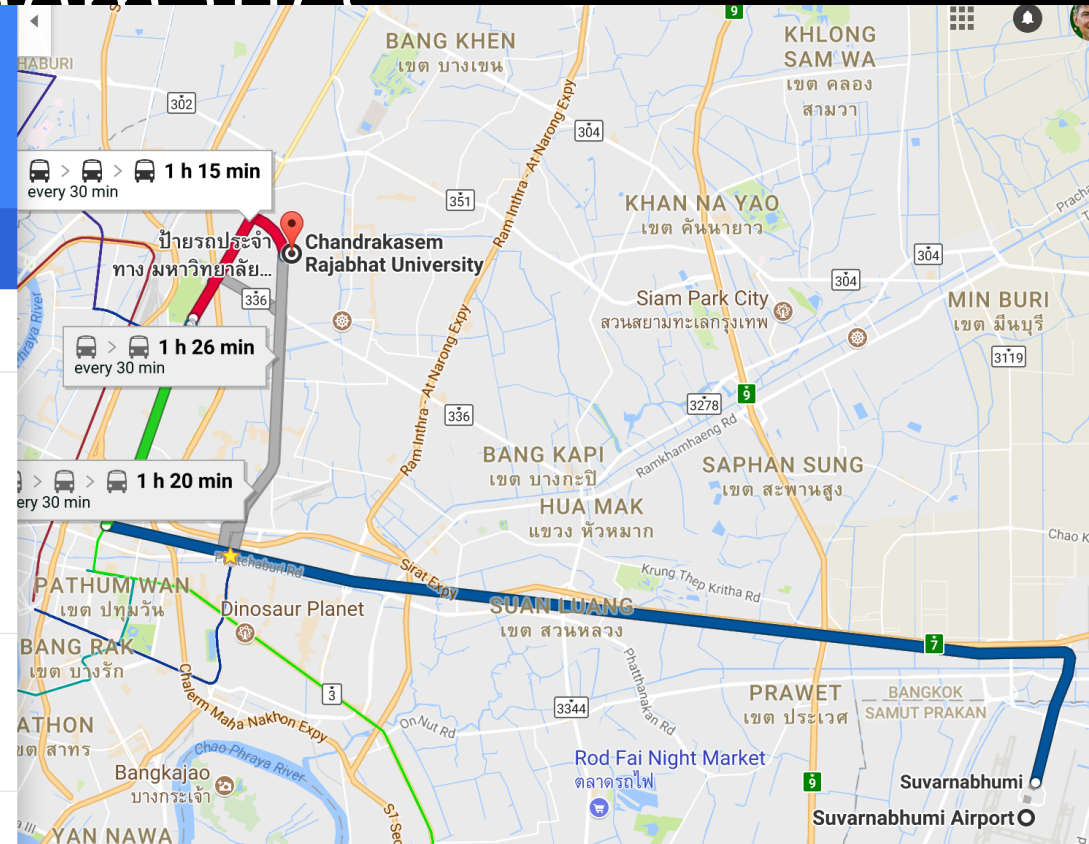
Applications of mathematics

S

Navigation app interface showing routes from Suvarnabhumi Airport to Chandrakasem Rajabhat University.

Routes and times:

- 3:29 PM–4:44 PM, 1 h 15 min (SRTET City Line, BTS Sukhumvit Line)
- 3:29 PM–4:55 PM, 1 h 26 min (SRTET City Line, BTS Sukhumvit Line)
- 3:29 PM–4:49 PM, 1 h 20 min



When people design, explain, or play with counting, measuring or locating, they are doing mathematics.

Potential contributions of math (basis for task/activity development)

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When people design, explain, or play,
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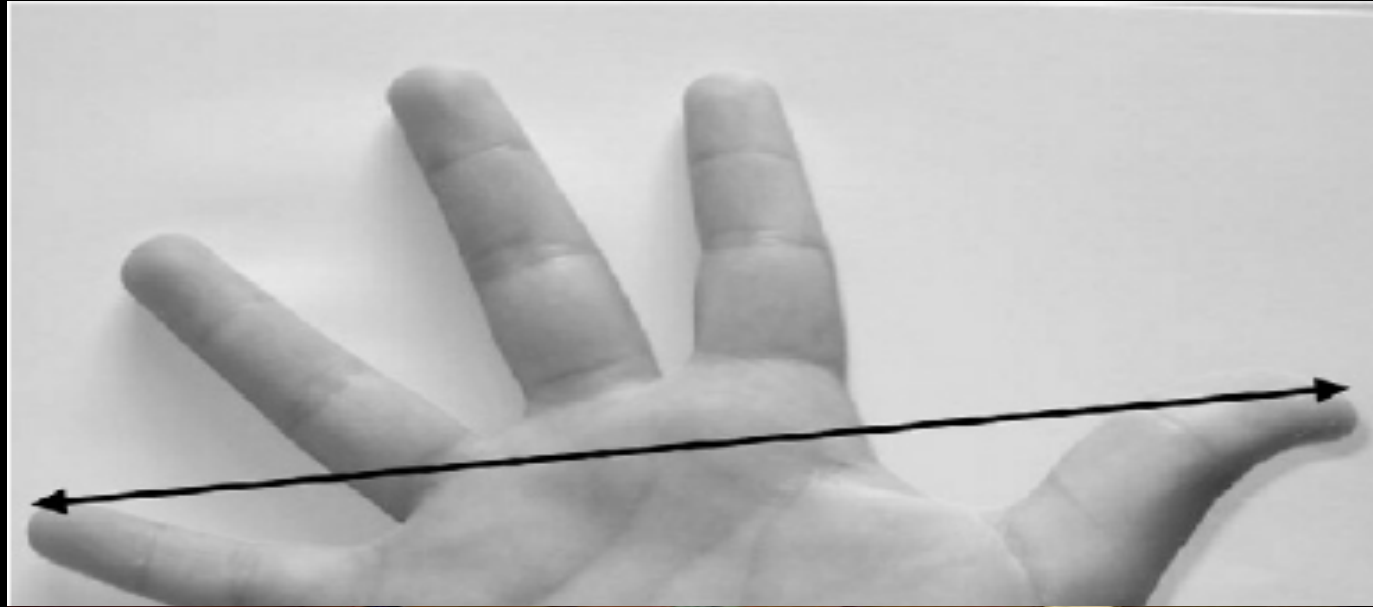


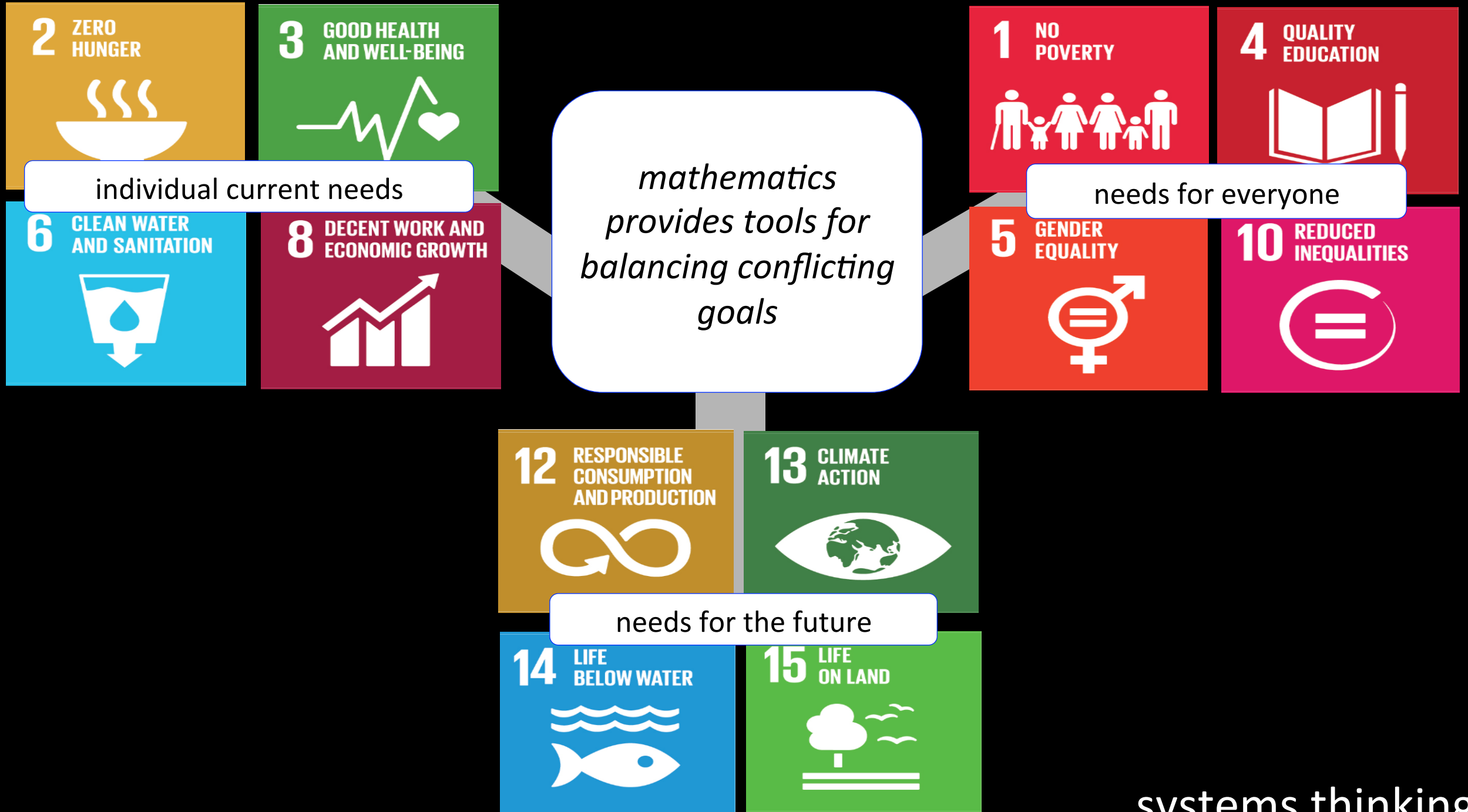
Potential contributions of mathematics

We can communicate with

- precision
- accuracy

Necessary for agreements
(treaties, contracts,...)





systems thinking

Potential contributions of mathematics

To balance conflicting needs we:

- consider measures that indicate identified goals,
- assign numeric values to things that are not easy to measure,
- weight these indicators to align with our values.

These indicators can

- measure various policy scenarios to identify an optimum decision,
- compare regions to see who/what needs attention.

This depends on the quality of our choices about:

- what to measure
- which measures to value the most, and
- the mathematical model for predicting results

It may seem arbitrary, but it is less arbitrary than the alternative.

Potential contributions of mathematics

popular press
(unclear method)

What is the
'best country'
in the world?

A new report ranks Germany as the best country in the world with Canada finishing second. The inaugural "Best Countries" survey ranks nations by looking at 75 factors such as sustainability, adventure, cultural influence, entrepreneurship and economic influence.

 1 Germany	 6 Australia
 2 Canada	 7 Japan
 3 United Kingdom	 8 France
 4 United States	 9 Netherlands
 5 Sweden	 10 Denmark

Source: U.S. News & World Report, BAV Consulting and Wharton School © Global News

GDP growth, HDI growth (human development index), IWI growth (inclusive wealth index)



GINI coefficient
(generalized inequality index)

Gini coefficient, after taxes and transfers ^[10]								
	Country	mid-70s	mid-80s	around 1990	mid-90s	around 2000	mid-2000s	Late 2000s
1	 Slovenia						0.246	0.236
2	 Denmark		0.221	0.226	0.215	0.226	0.232	0.248
3	 Norway		0.222		0.243	0.261	0.276	0.250
4	 Czech Republic			0.232	0.257	0.260	0.268	0.256
5	 Slovak Republic						0.268	0.257
6	 Belgium		0.274		0.287	0.289	0.271	0.259
7	 Finland	0.235	0.209		0.218	0.247	0.254	0.259
8	 Sweden	0.212	0.198	0.209	0.211	0.243	0.234	0.259
9	 Iceland		0.236		0.238	0.252	0.265	0.261
10	 Austria			0.273	0.294	0.293	0.291	0.272
11	 Portugal		0.247		0.259	0.261	0.258	0.288
12	 France		0.290	0.290	0.277	0.287	0.288	0.293
13	 Ireland		0.331		0.324	0.304	0.314	0.293
14	 Netherlands	0.283	0.272	0.292	0.297	0.292	0.284	0.294
15	 Germany		0.251	0.256	0.266	0.264	0.285	0.295
16	 Iceland						0.257	0.301
17	 Switzerland					0.279	0.276	0.303

mathematics



GROSS NATIONAL HAPPINESS

The essence of the philosophy of Gross National Happiness is the peace and happiness of our people and the security and sovereignty of the nation.



Gross national happiness (GNH vs GDP as guide for decisions)

HOME 2015 GNH SURVEY REPORT

GNH INDEX

GNH TOOLS

9 DOMAINS

2015 GNH SURVEY REPORT



A Compass Towards a Just and Harmonious Society

2015 GNH Survey Report

The Third Gross National Happiness (GNH) Survey was conducted from January to May 2015 with financial support from the Japan International Cooperation Agency (JICA) and the Royal Government of Bhutan.

SEARCH

LATEST NEWS

2015 GNH SURVEY REPORT

PROVISIONAL FINDINGS OF 2015 GNH SURVEY

SUMMARY OF 2015 GNH SURVEY FINDINGS

Rank	Country	Int\$	Year
1	Qatar	141,543	2015
—	Macau	111,497	2015
2	Luxembourg	101,926	2015
3	Singapore	85,382	2015
4	Brunei	78,369	2015
5	Kuwait	74,646	2015
6	United Arab Emirates	69,971	2015
7	Ireland	65,144	2015
8	Norway	61,197	2015
9	Switzerland	61,086	2015
—	Hong Kong	56,924	2015
10	United States	56,116	2015
11	Saudi Arabia	53,539	2015
—	Bermuda	52,436	2013
—	Cayman Islands	49,902	2011
12	Netherlands	48,313	2015
13	Austria	48,194	2015
14	Germany	47,377	2015
15	Sweden	46,704	2015

GDP per capita (or PPP)

Potential contributions of mathematics

We can measure:

- pollution
- income
- law breaking
- education attainment
- reading
- critical thinking
- ...

We can compare across:

- regions
- gender
- age
- ethnicity
- immigration status
- sexual identity
- socio-economics
- ...

This supports

- local and global decision making
- engaged citizens

Potential contributions of mathematics

We can measure:

- pollution
- income
- law breaking
- education attainment
- reading
- critical thinking
- ...

We can compare across:

- regions

Air Pollution in Asia



This supports

- local and global decision
- engaged citizens

Potential contributions of mathematics

Limitations:

- When is it appropriate to measure love, fear, pain, hunger, ...?
- Mathematics can support good motives and bad. (bombs, eugenics, ...)
- Abstraction ignores local knowledge.

When doing human mathematics,
students should discuss the limitations.

Potential contributions of mathematics



- Abstraction ignores local knowledge.

A photograph of a person looking at a large electronic display board showing stock market data. The board displays various stock prices and changes, with a person's head visible in the foreground looking at the screen.

日経平均株価 2466.94 +10.79					
「改良発売」安全運転システムを搭載					
で2割下げ 14:56 <NQN>◇<東証>					
今日の古い 提供: ネット証券					
東ソー	トクヤマ	デンカ	信越化	協和キリン	ヤフー
692	596	195	487	6352	1760
+13	+23	0	+17	+237	-65
683	580	193	473	6237	1777
695	602	197	492	6425	4280
667	579	191	473	6214	4140
HD	宇部興	日化薬	電通	花王	浜ゴム
5.6	210	1244	6250	54	2133
4.3	+2	+2	+130	54	+32
1.7	210	1232	6180	54	2089
9.2	212	1256	6290	54	2152
4.5	208	1224	6120	54	2058
塩野義	中外薬	エーザイ	第一三共	海力	TOTO
190	4380	3710	7052	3835	3835
-4	+105	+45	+22	+120	+120
195	4355	3765	7086	3810	3810

Mathematics is known for abstraction, but it has both processes:

- abstraction (moving to the abstract)
- particularization (moving from the general to the particular)

Potential contributions of mathematics

Moving away from context can help us see a system at work.

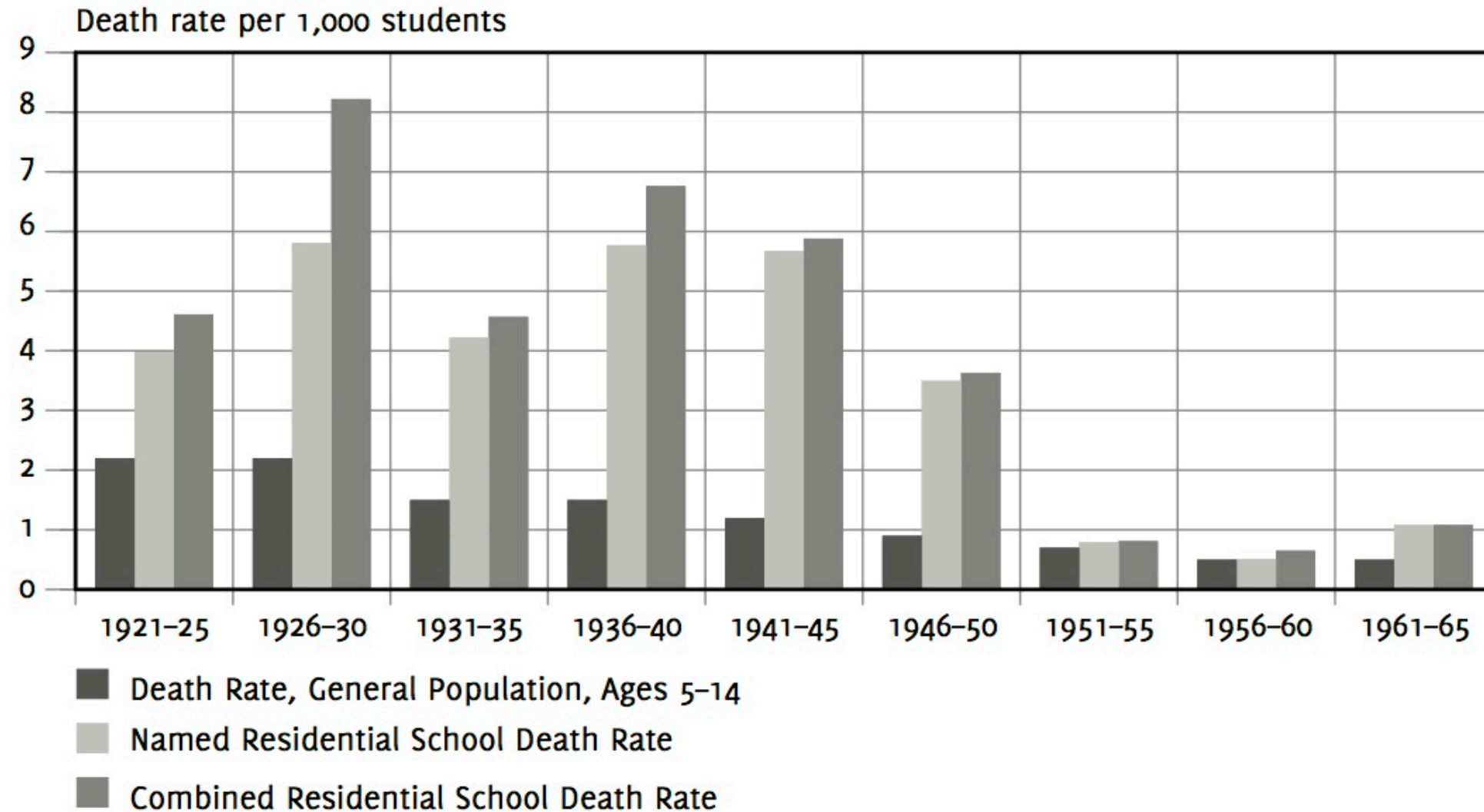
Incarceration Rates by Race 2010 (USA)

Race/ethnicity	% of US population	% of US population in prison	National incarceration rate (per 100,000)
White	64%	39%	450
Hispanic	16%	19%	831
Black	13%	40%	2,306

**Mathematics is known for abstraction,
but it has both processes:**

- abstraction (moving to the abstract)
- particularization (moving from the general to the particular)

Comparative death rates per 1,000 population, residential schools (Named and Unnamed registers combined) and the general Canadian population of school-aged children, using five-year averages from 1921 to 1965.



Source: Fraser, Vital Statistics and Health, Table B35-50, <http://www.statcan.gc.ca/pub/11-516-x/section-b/4147437-eng.htm>; Rosenthal, "Statistical Analysis of Deaths," 13.

Potential contributions of mathematics

Beauty

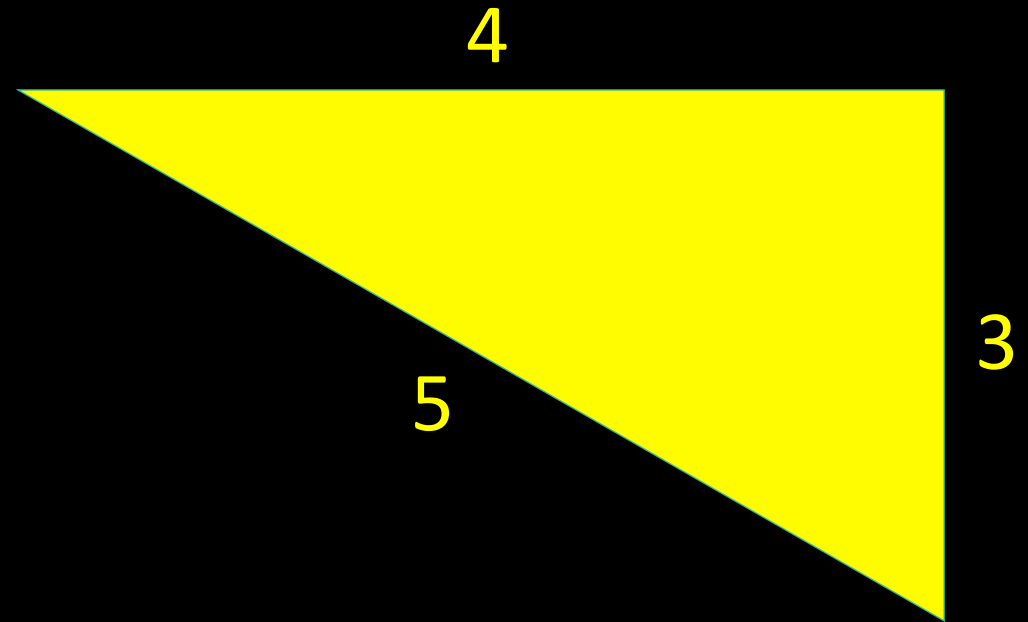
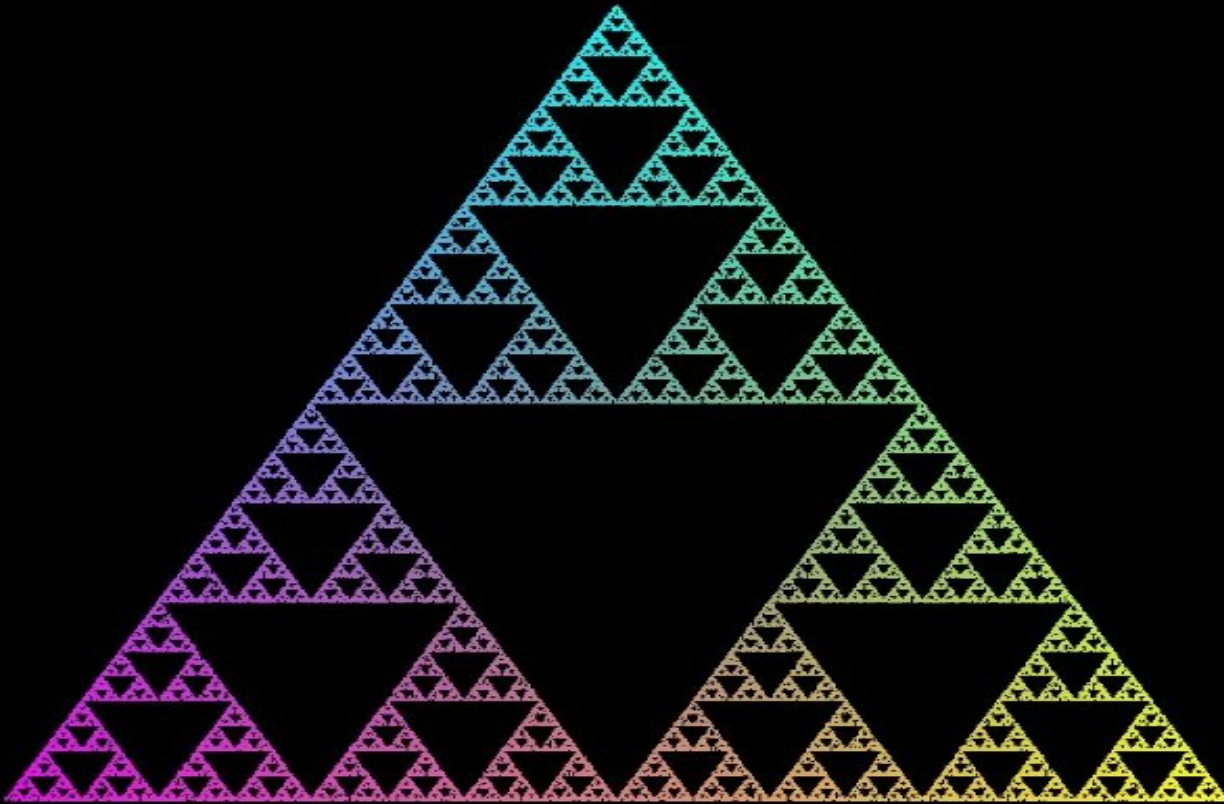
- appreciating diverse ingenuity
- standing beside each other to appreciate common experience



Potential contributions of mathematics

Beauty

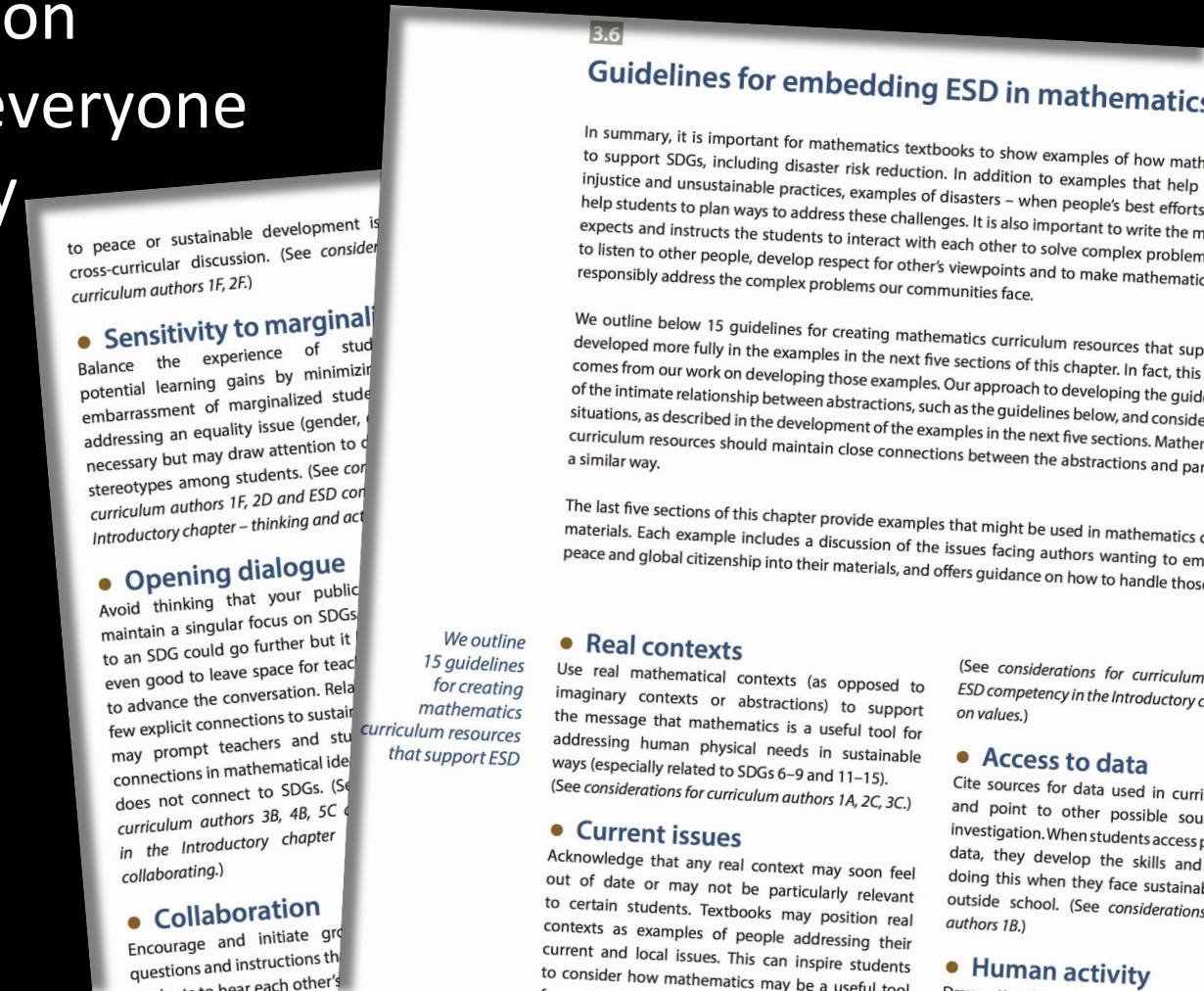
- appreciating diverse ingenuity
- standing beside each other to appreciate common experience



Tools for embedding sustainability/peace

Real contexts
Attention to context
Attention to human choices
Current issues
Complexity (avoid simplicity)
Acknowledging values
Access to data
Interdisciplinarity
Sensitivity to marginalization

Open dialogue
Collaboration
Access to everyone
Technology
Courage
Trust

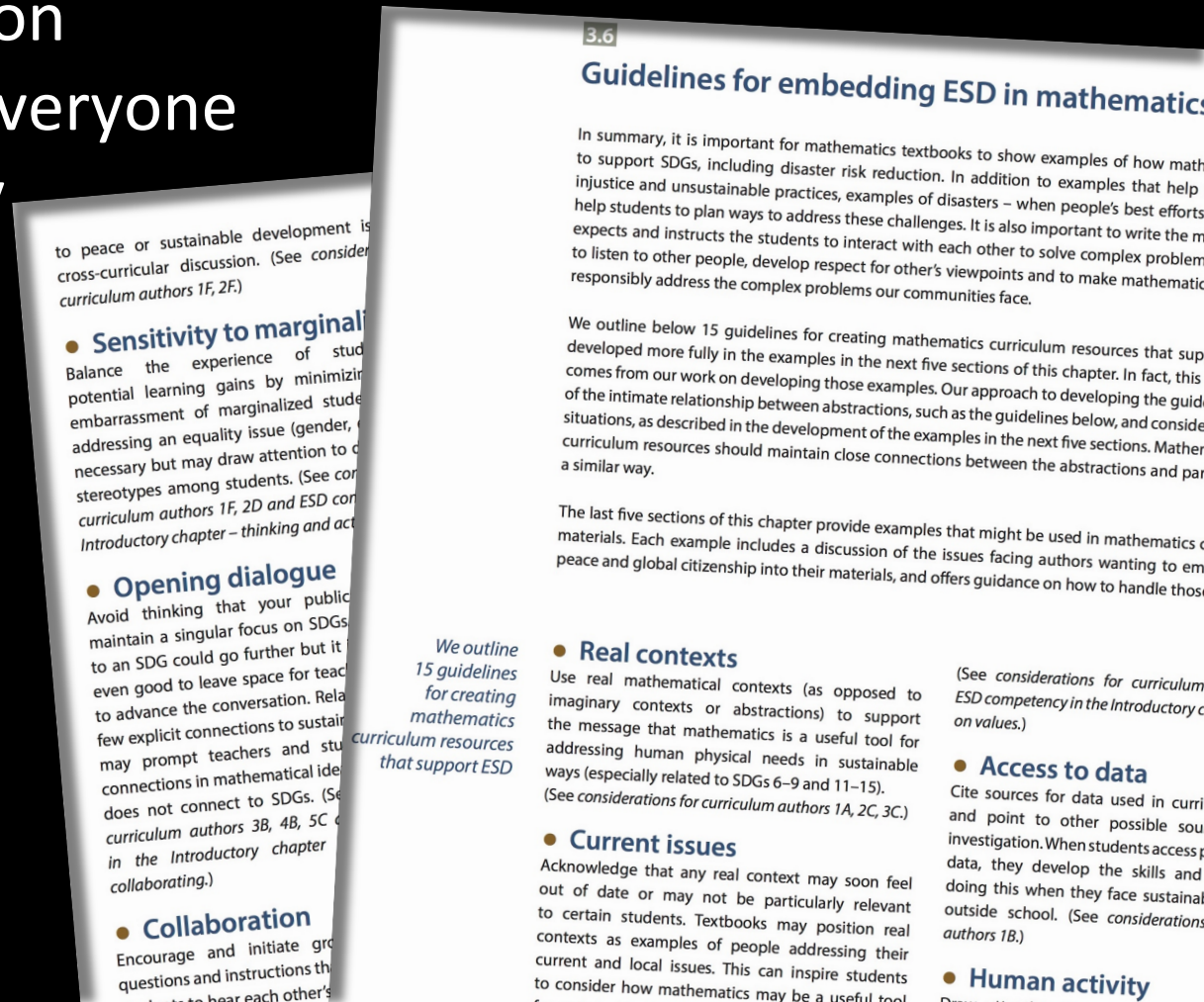


Tools for embedding sustainability/peace

e.g., Lesson study by Roberto & Pedro – CO₂ emissions in Peru and Chile

Real contexts
Attention to context
Attention to human choices
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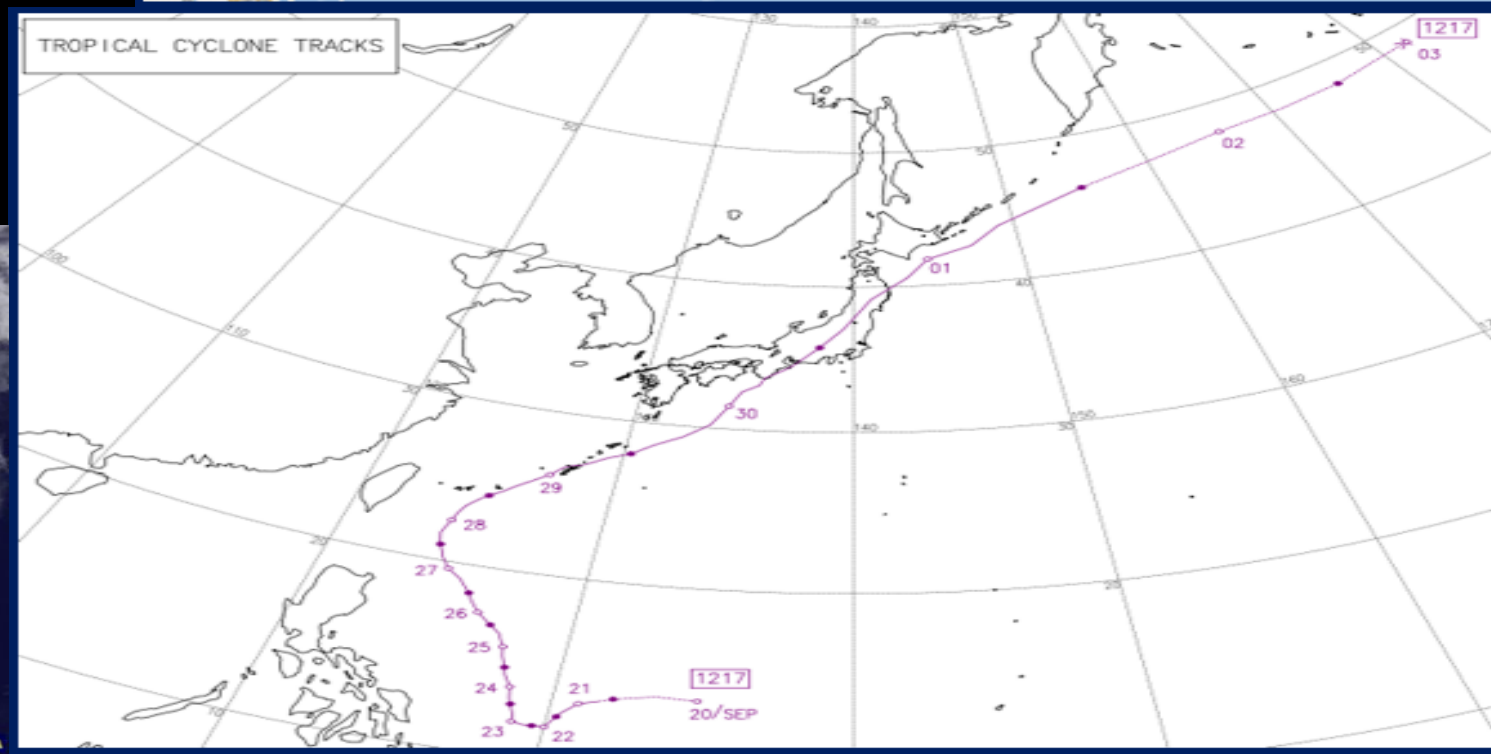
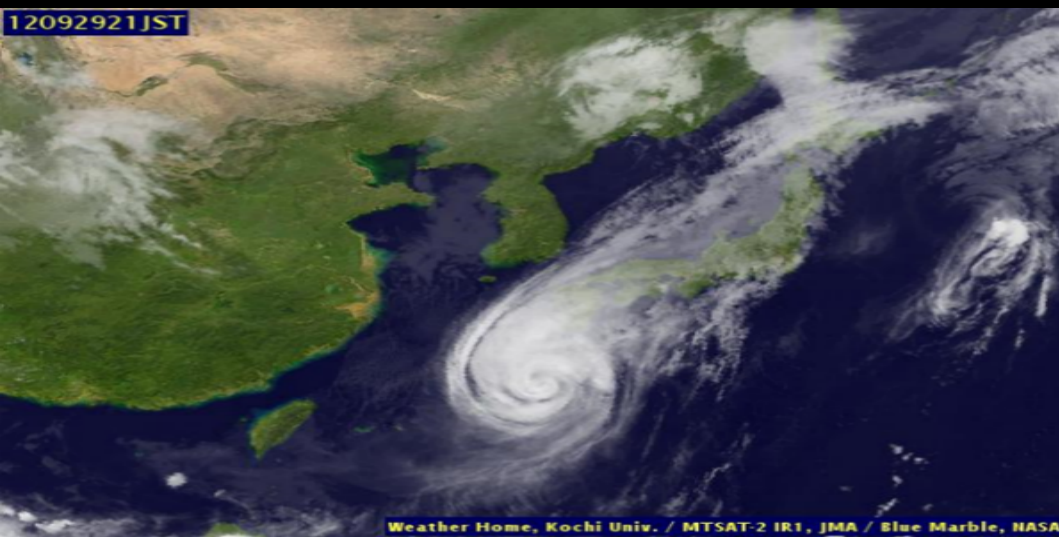


Tools for embedding sust

Real contexts

- contexts real to the student with real data
- shows that mathematics is useful

e.g. Isoda



Tools for embedding sustainability/peace

Attention to context

- Draw on students' context knowledge
- Structure textbooks around context instead of procedures
 - Chapters and topics organized around procedures tells students that the procedure is more important than the context.
 - Chapters and topics organized around contexts tells students that the mathematics responds to context.

Tools for embedding sustainability/peace

Attention to context

- Draw on students' context knowledge
- Structure textbooks around context instead of procedures
 - Chapters and topics organized around procedures tells students that the procedure is more important than the context.
 - Chapters and topics organized around contexts tells students that the mathematics responds to context.

Word Problems

- A long tradition
 - thus hard to change
- “throw-away contexts”

Children aged 14, 15, and 16 go to the store. Candy bars are on sale for 43¢ each. They buy a total of 12 candy bars. How much do they spend, not including tax? (Peterson & Gutstein)

Tools for embedding sustainability/peace

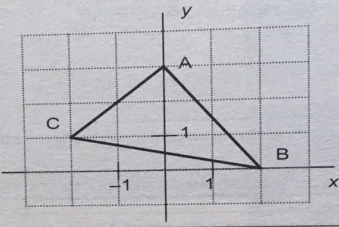
Attention to human choices

- Questions that have multiple possible responses
- Language that identifies human choice. (“I chose to”) (Wagner and Herbel-Eisenmann)
- People from diverse demographic groups so that students can visualize themselves using mathematics powerfully

Examples

Example 1 Using Mapping Notation to Translate a Shape

Determine the coordinates of the vertices of $\triangle ABC$ after it is translated with the mapping $(x, y) \rightarrow (x + 4, y - 2)$. Explain how you got your answer.



Solution

The vertices are:

A(0, 3)

B(2, 0)

C(-2, 1)

A(0, 3) $\rightarrow$ A'(0 + 4, 3 - 2),
or A'(4, 1)

B(2, 0) $\rightarrow$ B'(2 + 4, 0 - 2),
or B'(6, -2)

C(-2, 1) $\rightarrow$ C'(-2 + 4, 1 - 2),
or C'(2, -1)

Thinking

- I wrote the coordinates of the vertices of $\triangle ABC$.
- I substituted the x - and y -coordinates from each point into the mapping notation expression, $(x + 4, y - 2)$.



Solution 1

D(3, -1) $\rightarrow$ D'(-2, 6)

horizontally: $-2 - 3 = -5$

vertically: $6 - (-1) = 7$

Translation rule: $[-5, 7]$

E(2, -3) $\rightarrow$ E'(-3, 4)

Thinking

- I chose to use point D and its image D' to figure out how $\triangle DEF$ was translated, because I knew each point in $\triangle DEF$ translates the same way.

- I subtracted the corresponding x -values and the corresponding y -values to find the length of the horizontal slide and the vertical slide. (I made sure I subtracted the y -values in the same order as I did the x -values, $D' - D$.)

- I described the translation using a translation rule.



Solution 2

E(2, -3) $\rightarrow$ E'(-3, 4)

horizontally: $-3 - 2 = -5$

vertically: $4 - (-3) = 7$

Mapping notation:
 $(x, y) \rightarrow (x - 5, y + 7)$

Thinking

- I chose point E and its image E' to figure out how $\triangle DEF$ was translated, because I knew each point in $\triangle DEF$ translates the same way.

- I subtracted the x -value in E from the x -value in E' to find the length of the slide horizontally.

- I subtracted the y -values in E from the y -value in E' to find the length of the slide vertically.

- I described the translation using mapping notation.



If we think about the choices in our mathematics, it helps us

STOP

to think about the choices behind the math others do

Example 2 Representing a Translation

The vertices of $\triangle DEF$ are D(3, -1), E(2, -3), and F(1, -1).

The vertices of $\triangle D'E'F'$ are D'(-2, 6), E'(-3, 4), and F'(-4, 6).

Describe the translation that maps $\triangle DEF$ onto $\triangle D'E'F'$.

Tools for embedding sustainability/peace

Complexity (avoid simplicity)

- Simplifying context gives the idea that the procedure is more important than the contexts it is designed to address
or that math is not able to handle complexity

- Systems thinking



Tools for embedding sustainability/peace

Acknowledge values

- Doing a problem requires decisions about what is most important
- Any choice of context suggests that the topic is an important one for students.

For example:

A problem about running a business or responding to climate change.

- If it is about running a business, can a student choose to value profit, sustainability, or ... ?
- If it is about climate change, can a student choose how to balance current needs with future needs?

Tools for embedding sustainability/peace

Access to data

- Cite sources for data used.
- Point to other possible sources for further investigation.

When students access publicly available data, they develop the skills and confidence for doing this when they face sustainability challenges outside school.

Tools for embedding sustainability/peace

Interdisciplinarity

- Peace and sustainable development problems are especially good for interdisciplinary connections.
- Making interdisciplinary connections values complexity of contexts, and invites discussion about how mathematics can help with decisions in such complex situations.

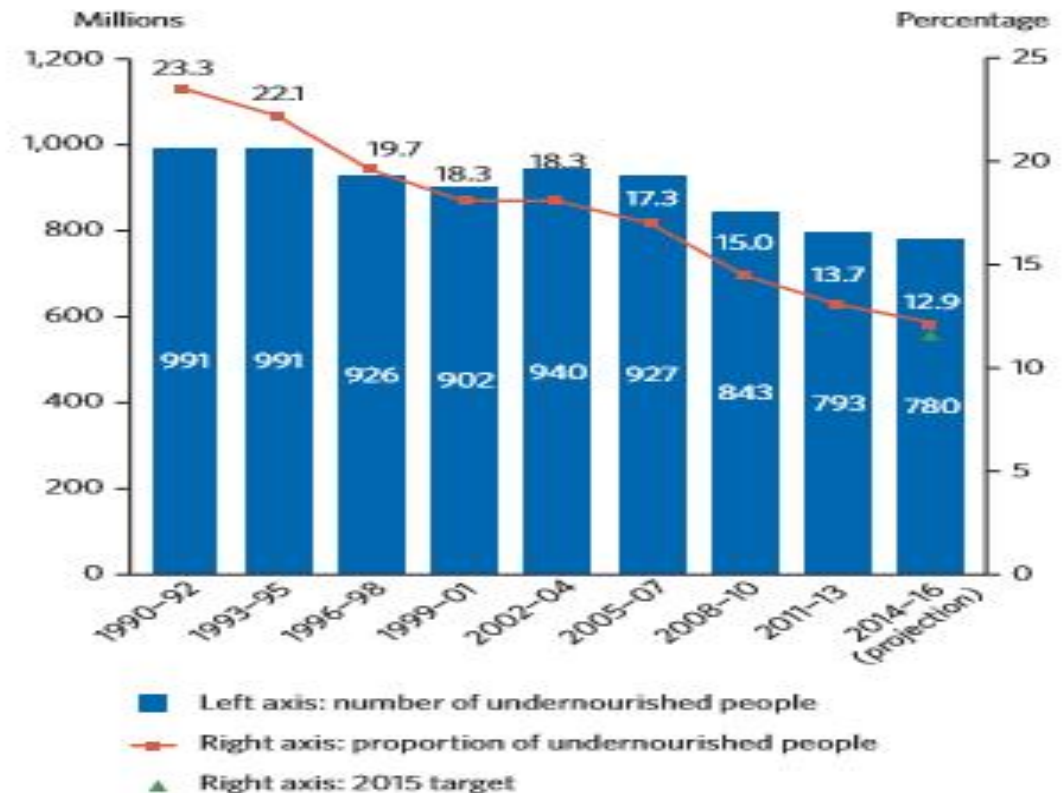
Tools for embedding sustainability/peace

Sensitivity to marginalization

- Directly addressing an equality issue (gender, ethnic, etc.) is necessary but may draw attention to disparities and stereotypes among students.

e.g., We should give problems that ask about income distribution. Students who are very rich or very poor might be embarrassed.

Number and proportion of undernourished people in the developing regions, from 1990–1992 to 2014–2016



Tools for embedding sustainability/peace

Collaboration

- Encourage and expect group work.
- This enables students to hear each other's ideas and so
 - they learn to value other ideas
 - they see that mathematics involves choices

What information does each form of representation show especially well?

versus

news article. Describe how your group decided which representation is the best choice.

Tools for embedding sustainability/peace

Technology

- Get students using their technology to gather information.
- More than finding things on the internet
- Smart phones can collect and map data in a community.
 - GPS
 - seismometers
 - apps that analyze sound
 - ...

e.g., Rubel, Hall-Wieckert, and Lim, 2016



Tools for embedding sustainability/peace

Courage

- Sustainable development questions are usually politically sensitive
- These are the most important issues

If we avoid politically sensitive issues:

- We are saying they are less important.
- We are saying that mathematics cannot be used for them.

Tools for embedding sustainability/peace

Trust

When the problem is politically sensitive...

- Let students and teachers make judgments based on their mathematical investigation and knowledge of the relevant contexts (Do not make the judgments in the text)

Teachers and students may not be completely ready to handle these difficult issues.

- This is how they learn to handle them.
- No one is completely ready.

A teacher's guide may be helpful.

Mathematics: proof and justification

History tells us that quantification is associated with the development of democracies and other forms of community responsibility (Porter, 1995).

When decision makers are not identified through heredity or connection to deities, they must prove that their decisions are made in the interests of the collective good.

- Mathematics gives citizens tools for this kind of justification.

Mathematics textbooks for sustainable development

Thank you

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Photo/chart sources:

http://archive.archaeology.org/1111/artifact/wittenberg_germany_tally_stick.html

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